

PAPR Reduction in OFDM & MC-CDMA System using Nonlinear Companding Techniques

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Abstract - High peak to average power ratio (PAPR) of the transmitted signal is a major drawback of OFDM and MC-CDMA systems. In this paper various existing nonlinear companding transforms are analyzed and compared for the reduction of peak to average power ratio in OFDM and MC-CDMA systems. Nonlinear companding transforms transform the amplitude or power of the original signals into uniform distribution, which can effectively reduce the PAPR for different modulation formats and subcarrier sizes without any complexity increase and bandwidth expansion. Nonlinear companding technique adjust both large and small signals and can keep the average power at the same level. Nonlinear companding transforms can significantly improve the performance of OFDM and MC-CDMA system including bit-error-rate and PAPR reduction.

Index Terms –Nonlinear companding transform, peak to average power ratio (PAPR), orthogonal frequency division multiplexing (OFDM), Multicarrier code division multiple access (MC-CDMA)

1.INTRODUCTION

As a multicarrier modulation technique, Orthogonal Frequency Division Multiplexing (OFDM) [1] and Multicarrier Code Division Multiple Access (MC-CDMA) [2] has the following advantages: (1) robust to multi-path fading, inter-symbol interference, co-channel interference and impulsive parasitic noise; (2) lower implementation complexity compared with the single carrier solution; and (3) high spectral efficiency in supporting broadband wireless communications. Therefore, OFDM is believed to be a suitable technique for broadband wireless communications and has been used in many wireless standards, such as Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T), Wireless Local Area Networks (WLAN). Multicarrier Code Division Multiple Access (MC-CDMA), which is a combination of two radio access techniques called Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA), has attracted more and more attentions as a very promising modulation technique [3].

The main idea behind MC-CDMA is to spread and convert input signals into parallel data streams, which are then transmitted over multiple carriers. MC-CDMA can realize high bit rate and large capacity transmission.

Original OFDM and MC-CDMA signals have very high Peak-to-Average Power Ratio (PAPR), which require sophisticated (expensive) radio transmitters with their high power amplifiers operating in a very large linear range otherwise; nonlinear signal distortion occurs and leads to high adjacent channel interference and poor system performance [2].

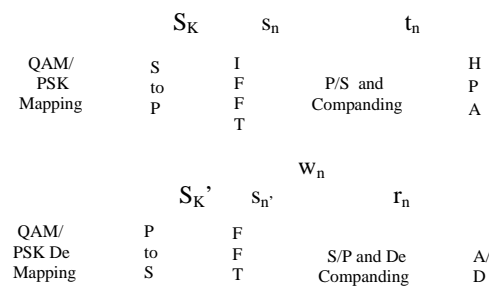


Figure 1. OFDM system using non linear companding technique

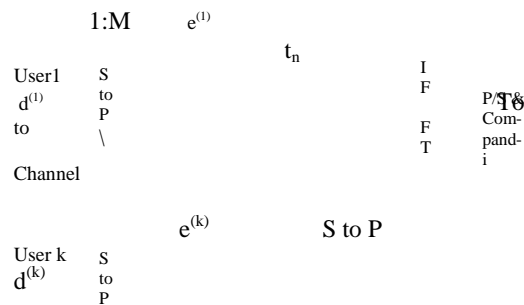


Figure 2. MC-CDMA system using non linear companding technique

Recently, various schemes have been proposed to reduce the PAPR of OFDM and MC-CDMA signals [4-11]. Among these schemes, nonlinear companding transforms are the most attractive schemes due to their good system performance, the simplicity of implementation, without restriction on the number of subcarriers, the type of constellation and any bandwidth expansion [12-15].

In this paper, we analyze three nonlinear companding techniques, namely “C1(.)”, “C2(.)”[14] and “exponential companding- C(.)”, [15], to reduce the PAPR of

OFDM signals. These techniques effectively transform the original Gaussian-distributed OFDM signals into uniform-distributed (companded) signals without changing the average power level. Unlike the μ -law companding scheme, which mainly focuses on enlarging small signals, nonlinear companding schemes adjust both small and large signals without bias so that it is able to offer better performance in terms of PAPR reduction, Bit-Error-Rate (BER) and phase error for OFDM and MC-CDMA systems. The rest of this paper is organized as follows.

In Section II, typical OFDM and MC-CDMA system is given and the high PAPR problem is formulated. Then, nonlinear companding techniques, namely “C1(.)”, “C2(.)” and “exponential companding-”C(.),” is analyzed in Section III to reduce PAPR. In Section IV, the performance of the three companding schemes are studied and compared through computer simulations, followed by conclusions in Section V.

II. PROBLEM FORMULATION

Figure 1&2 shows the block diagram [14], [10] of a typical OFDM and MC-CDMA systems using the nonlinear companding technique for PAPR reduction. Let N denote the number of sub-carriers used for parallel information transmission and let $S_k (0 \leq k \leq N-1)$ denote the k^{th} complex modulated symbol in a block of N information symbols. The outputs s_n of the N -point Inverse Fast Fourier Transform (IFFT) of S_k are the OFDM signal samples over one symbol interval, or mathematically

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(\frac{j \cdot 2\pi kn}{N}\right) \quad (1)$$

According to the central limit theorem, it follows that for large values of N , s_n becomes Gaussian distribution with the probability density function (PDF)

$$f_{s_n}(s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad (2)$$

where σ^2 is the variance of the original MCM signals.

Therefore, the signal s_n has distribution with the cumulative distribution function (CDF) as following

$$F_{s_n}(s) = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{s}{\sqrt{2}\sigma}\right)\right) \quad (3)$$

where $\operatorname{erf}(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-y^2} dy$.

Then, the PAPR of MCM signals s_n in one symbol period is defined as

$$\text{PAPR}(s_n) = 10 \log \frac{\text{Max}\{|s_n|^2\}}{\sigma^2} \text{ (dB)} \quad (4)$$

By using companding transform, s_n are companded before converted into analog waveforms and amplified by the HPAs. $sc(n)$ is given by $sc(n) = C(s_n)$, ($n = 0, 1, \dots, N-1$),

where $C(\cdot)$ is the companding transform function.

Then, OFDM and MC-CDMA signals are transmitted into the radio channel. After passing through both AWGN and the frequency selective fading channel, the received signal can be formulated as $r(n) = sc(n) + w(n) = C(s_n) + wn$, where $w_n = w(n) \cdot \text{hinv}(n)$ with the variance of $\sigma^2 w$. At the received end of the de-companded MCM system, $r(n)$ has to be expanded according to $S_n' = C^{-1}(sc(n) + w_n) = s_n + C^{-1}(w_n)$, where $C^{-1}(\cdot)$ is the de-companding function, or the inverse function of $C(\cdot)$.

III. ANALYSIS OF NONLINEAR COMPANDING TRANSFORMS

In this section, we propose the design criteria of the nonlinear companding transforms, which is based on derivation and analysis of some existing novel nonlinear companding transforms [14], [15]. As examples, the existing nonlinear companding transforms can effectively reduce the PAPR of the OFDM signals by transforming the statistics of the amplitudes or power of the original OFDM signals into uniform distribution. These novel schemes also have the advantage of maintaining a constant average power level in the nonlinear companding operation. The strict linearity requirements on HPA can then be partially relieved.

Let us denote X and Y as random variables representing the amplitudes of the inputs and outputs signals of the companding transform $C1(\cdot)$ with the CDFs marked $F_x(x)$ and $F_y(y)$, respectively. Since Y is to be desired the uniform distribution in the interval $[0, h1]$ ($h1 > 0$), then the CDFs of Y can be given by

$$F_y(y) = \frac{y}{2h1} + \frac{1}{2}, \quad 0 \leq y \leq h1 \quad (5)$$

Since the $F_x(x)$ and $F_y(y)$ are strictly monotone increasing functions, they have corresponding inverse functions. At the same time, the companding transform $C1(\cdot)$ is also restricted to be a strictly monotone increasing function and has its inverse transform. When these conditions are satisfied, we can deduce these conclusions as following

$$\begin{aligned} F_x(x) &= \text{Prob}\{X \leq x\} \\ &= \text{Prob}\{C1(X) \leq C1(x)\} \\ &= F_y(C1(x)) \end{aligned} \quad (6)$$

Therefore, the companding function $C1(\cdot)$ is given by the following identity

$$C1(x) = F_y^{-1}[F_x(x)] \quad (7)$$

Substituting (3) and (5) into (7) shows that

$$C1(x) = h1 \cdot \text{erf}\left(\frac{x}{\sqrt{2}\sigma}\right), 0 \leq x \leq 1 \quad (8)$$

The positive constant $h1$ determines the average power of the output signals. In order to keep the input and output signals at the same average power level during the companding transform, companding transform $C1(\cdot)$ should satisfy $E[|sn|2] = E[|tn|2]$. Therefore, we can obtain

$$\sigma^2 = \int_{-h1}^{h1} (tn)^2 p(tn) dtn \quad (9)$$

Substituting (5) into (9), we can obtain that $h1 = \sqrt{3}\sigma$. Therefore, nonlinear companding transform function $C1(x)$ can be expressed as

$$C1(x) = \sqrt{3} \sigma \cdot \text{erf}\left(\frac{x}{\sqrt{2}\sigma}\right), 0 \leq x \leq 1 \quad (10)$$

Considering the phase of input OFDM signals, we can obtain another nonlinear companding transform function based on transforming the power of MCM signals into a uniform distribution as following

$$C2(x) = \text{sgn}(x) \cdot \sqrt{h2 \cdot \text{erf}\left(\frac{|x|}{\sqrt{2}\sigma}\right)} \quad (11)$$

Similarly, in order to keep the average power of the companded OFDM signals the same level with that of the original OFDM signals, namely $E[|sn|2] = E[|tn|2]$, and thus, the parameter $h2$ can be obtained $h2 = 2\sigma$. Therefore, $C2(x)$ can be expressed as

$$C2(x) = \text{sgn}(x) \cdot \sqrt{2\sigma \cdot \text{erf}\left(\frac{|x|}{\sqrt{2}\sigma}\right)} \quad (12)$$

In the same way, the nonlinear transform function $C(x)$ can be derived

$$C(x) = \sqrt{6}\sigma [1 - \exp\left(\frac{-x^2}{2\sigma^2}\right)] \quad (13)$$

It belongs to the type of “Exponential Companding Transform”[15]. Considering the phase of input signals, the exponential companding function $C(x)$ is given by

$$C(x) = \text{sgn}(x) \sqrt{\alpha [1 - \exp\left(\frac{-x^2}{\sigma^2}\right)]} \quad (14)$$

where $\text{sgn}(x)$ is the sign function. The positive constant α determines the average power of output signals. The companded signals have uniformly distributed amplitudes and power, respectively for the cases $d=1$ and $d=2$. At the receiver side, the inverse function of $C(x)$ is used in the de-companding operation.

Figure. 2(a) depicts the profiles of three types of nonlinear companding transforms $C1(\cdot)$, $C2(\cdot)$ and μ -law, from that we can see the proposed nonlinear companding

transforms can compress large signals and expand small signals simultaneously. However, μ -law transform only enlarges small signals and does not change the signal peaks, which increases average power of the companded signals.

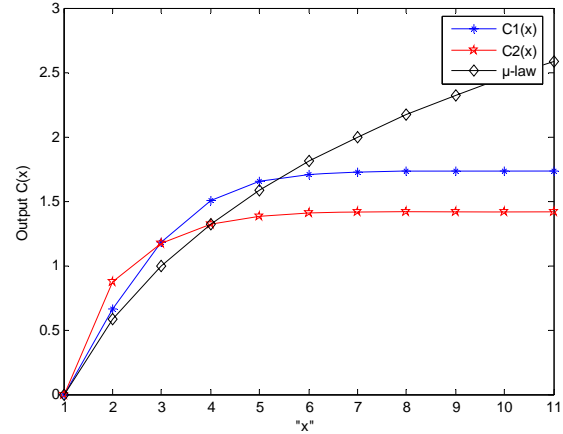


Figure .2 (a) Profiles of $C1(\cdot)$, $C2(\cdot)$ and μ -law

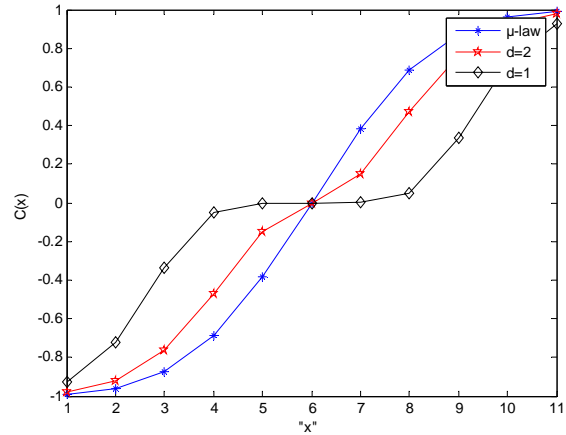


Figure .2 (b) The exponential companding function $C(x)$

Figure 2(b) shows exponential companding function $C(x)$ with degree ‘ d ’ as a parameter. The exponent ‘ d ’ is called the degree of a specific exponential companding scheme. The companded signals have uniformly distributed amplitudes and powers, respectively, for the cases $d=1$ and $d=2$. When $d \geq 2$, the proposed function can compress large input signals and expand small signals simultaneously.

IV.PERFORMANCE EVALUATION

To verify the performance of the nonlinear companding schemes in the PAPR reduction and BER performance, the simulation parameters are set as follows. For OFDM system: BPSK modulation, the number of data symbols per user N is 8, 16, 32 & 64. For MC-CDMA system: BPSK modulation, the number of data symbols per user N is 8 and 16, the number of active users K is 2 and

3 . Hadamard transform is used as spreading code with length $L=8$. In order to obtain sufficient transmit power; most radio systems often employ HPAs in the transmitter.

TABLE I
OFDM PAPR REDUCTION COMPARISON

N	PAPR(db)					
	Ideal	C1(.)	C2(.)	C(.) d=1	C(.) d=2	μ -law
8	3.010	1.906	1.491	1.986	2.128	2.754
16	2.916	2.119	1.403	2.185	2.433	2.701
32	5.016	3.920	2.589	4.014	4.376	4.676
64	8.304	6.683	4.151	6.826	6.983	7.762

PAPR reduction comparison of OFDM for various companding transforms are given in Table I. For different sub carrier sizes, the reduction in PAPR is calculated for four companding transforms C1(.), C2(.), C(.) and μ -law. Ideal value of PAPR is calculated without applying any companding transform. The reduction in PAPR increases with the increase in number of sub carriers. For various subcarrier sizes ($N = 8, 16, 32$ and 64) companding transform C2(.) gives the best reduction in PAPR. The PAPR value calculated for $N=64$, BPSK shows a reduction of 1.621db can be obtained for C1(.) and a reduction of 4.153db can be obtained for C2(.) and a reduction of 1.478 db can be obtained for C(.), exponential companding for $d=1$. Obviously, the signals companded by the nonlinear companding transforms C(.), C1(.), C2(.) can reduce the PAPR greater than that of μ -law companding transform.

TABLE II
2 USER MC-CDMA PAPR REDUCTION COMPARISON

N	PAPR(db)					
	Original	C1(.)	C2(.)	C(.) d=1	C(.) d=2	μ -law
8	12.4	6.44	4.74	6.65	6.55	11.1
16	9.40	3.26	1.72	3.46	3.25	8.03
N	PAPR(db)					
	Ideal	C1(.)	C2(.)	C(.) d=1	C(.) d=2	μ -law
8	10.7	5.34	4.12	5.50	5.24	9.57
16	7.68	2.13	0.96	2.28	1.99	6.46

TABLE III
3 USER MC-CDMA PAPR REDUCTION COMPARISON

PAPR reduction comparison of MC-CDMA for various companding transforms are given in Table II & III. The companding transform C2(.) gives the best reduction in PAPR among all other transforms for 2 user and 3 user MC-CDMA.

The wireless channel is assumed to be AWGN. Figure 3 & 4 depicts the performance of BER versus signal-to-noise ratio (SNR) of actual signals under different companding schemes. Note that, the performance bound

curve marked with "Ideal" is obtained without nonlinear transform and ignoring the effect of SSPA, which means directly transmitting the original MCM signals into channel. It has the best BER performance, but it has an extremely high PAPR compared with that of companded signals.

Moreover, from Figure. 3 for OFDM system, compared to all companding schemes, C1(.) can offer better BER performance, and it is only about 4.0dB loss compared to the case without any companding scheme at $BER = 10^{-2}$. C1(.) can obtain an improvement of BER performance about 3.0 dB relative to C2(.), but it has higher PAPR than that of C2(.).

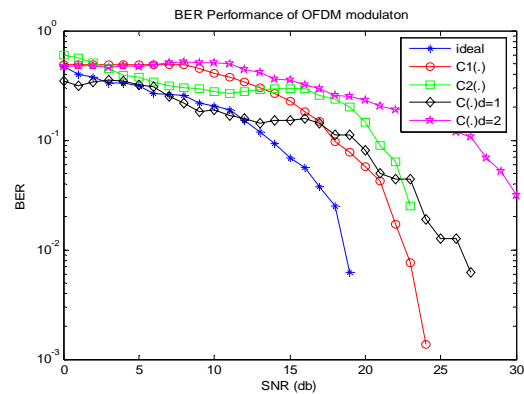


Figure .3. BER versus SNR of OFDM in AWGN fading channel under different companding transforms

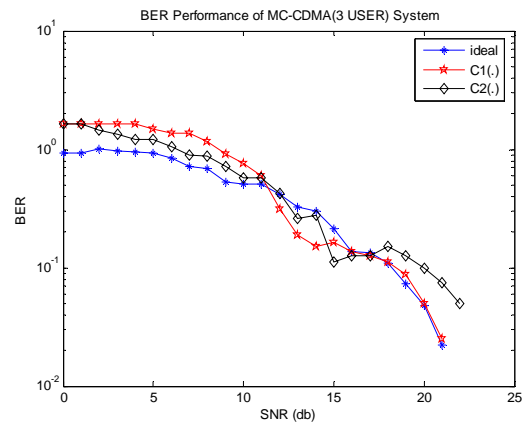


Figure 4. BER versus SNR of MC-CDMA in AWGN fading channel under different companding transforms

From Figure 4 for MC-CDMA system, compared to all companding schemes, C1(.) can offer better BER performance, and it is only about 0.12dB loss compared to the case without any companding scheme at $BER = 10^{-2}$. C1(.) can obtain an improvement of BER performance about 3.0 dB relative to C2(.), but it has higher PAPR than that of C2(.).

Due to the high PAPR, ideal OFDM&MC-CDMA signals have a very sharp, rectangular-like power spectrum [14]. This good property will be affected by the PAPR reduction schemes. Non linear companding scheme has

much less impact on the original power spectrum [15]. the major reason that the non linear companding scheme not only enlarges the small amplitude signals but also compresses the large amplitude signals, while maintains the average power unchanged by properly choosing parameters, which can increase the immunity of small amplitude signals from noise. The signals companded by $C1(\cdot)$ have a good spectrum characteristic and have almost no spectral regrowth caused by the PAPR reduction. For $C2(\cdot)$ nonlinear transform[14], it has the worst power spectrum, which may be caused by transforming the power not amplitude of OFDM&MC-CDMA into a uniform distribution, so that it can bring out-of-band distortion and result in more severe inter-carrier interference.

V.CONCLUSION

Non-linear companding transform is an effective technique in reducing the PAPR of OFDM&MC-CDMA signals. In addition, the schemes based on nonlinear companding techniques have low implementation complexity and no constraint on modulation format and sub-carrier size. In this paper, three nonlinear companding transforms based on the derivation and theoretical analysis of some existing nonlinear companding transforms were analyzed and evaluated for reducing the PAPR of OFDM&MC-CDMA signals. The BER performance of the nonlinear companding transforms was also studied by mat lab simulation. It is proved that the best tradeoff between BER performance and PAPR reduction can be achieved by $C1(\cdot)$ among these nonlinear transforms but $C2(\cdot)$ has the best PAPR reduction. Simulation results have shown that $C1(\cdot)$ companding scheme could offer better system performance in terms of PAPR reduction, power spectrum, BER for MC-CDMA system than OFDM system.

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